Plume structure and dynamics from thermocouple and spectrometer measurements

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ABSTRACT

The movements, structure, and dynamics of a heated stack plume are revealed by thermocouple and spectrometer measurements. A three-dimensional thermocouple array provides temperature data at two-second intervals. An animated display allows the temperature and position variations of a plume to be visualized, and demonstrates that the plume parameters have significant temporal variation at positions more than a few feet from the stack. Plots of temperature vs. downstream position show the transition from the "near field" to the "far field" regime. For a sideways-directed momentum plume, the temperature varies as the reciprocal of the downstream position. These results are consistent with published data and with theoretical expectations. Spectrometer data suggest that the shape of the 4.26-micron CO_2 transition depends on radial position in the plume. Spectra taken near the plume edge are relatively flat-topped, whereas measurements taken near the centerline show a line reversal due to absorption. These results are consistent with a plume structure involving a hot, optically thick core surrounded by an envelope of cooler gas.

Keywords: Gas plumes; effluent detection; infrared spectrometry; emissive; remote sensing

1. INTRODUCTION

For several years, the authors have been involved in efforts to provide improved understanding of gas plumes through application of infrared hyperspectral technology. This work involves systematic studies of plumes with known, controllable composition, parameters, and background conditions. Thermocouple arrays provide temperature data to complement spectroradiometer measurements. An emphasis on plume physics, including molecular spectroscopy and gas dynamics, allows as much information as possible to be extracted from the experimental measurements.

This paper discusses results relevant to plume dynamics and thermal structure. The importance of random fluctuations in plume parameters is demonstrated. The validity of simple relationships between plume parameters and downstream position is discussed for the case of a horizontally-directed plume. The spectral shape of the 4.26 μ m CO₂ emission band is related to plume thermal structure. The experimental results are verified by use of an optical transfer model. A companion paper discusses how plume temperature can be deduced from measurements of optically thin molecular transitions.¹

2. EXPERIMENTAL ARRANGEMENTS

2.1. Plume Source

The spectral data discussed in this paper were collected using a plume from a portable propane-burning plume generator. Spectra were collected on a runway at the Mid-Way Regional Airport near Waxahachie, Texas from April 6 to April 11, 2003. The plume generator was constructed and operated by AeroSurvey, Inc. Its base unit contains a centrifugal blower that feeds a combustion plenum, in which resides a liquid propane burner. Heated gas from the combustion plenum is fed into an elbow that redirects the gas into a 19-in. diameter stack. Plume temperature is regulated by adjusting the propane delivery pressure to the burner.

In an earlier (2002) series of measurements at Waxahachie, the stack was oriented vertically.² However, for the 2003 measurements discussed in this paper, the stack was operated in a horizontal configuration, with the plume being ejected along the wind direction. This configuration provided a sideways-directed plume, sometimes referred to as a

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"momentum jet" since the plume dynamics are initially dominated by the momentum with which the gas is ejected from the stack. The horizontal configuration provided easy access to the plume. In particular, it allowed the spectrometer to be placed near the plume, minimizing absorption effects from intervening atmosphere. Figure 1 shows the portable plume generator in its horizontal configuration.



Figure 1. Portable plume generator with horizontal stack configuration. Thermocouple array is on right side of figure.

2.2. Instrumentation

The spectral measurements were made using a Designs and Prototypes (D&P), Inc. MicroFTIR Model 101 spectroradiometer. Useful spectral data were obtained in the range from 3 to 18 μ m. The spectral resolution at 10 μ m was approximately 40 nm. The single scan time was 16 seconds.

Thermocouple readings were taken from the Waxahachie plumes using a large three-dimensional thermocouple array, as seen in Figure 1. The thermocouples were positioned on a wire grid that was supported by a 2 m x 2 m PVC framework. Two planes of thermocouples were used, giving readings at two downstream positions. Electrical outputs from the thermocouples were transmitted to daisy-chained solid-state modules, and then to a computer for automatic recording. A fresh set of readings from all the thermocouples was recorded every two seconds.

3. PLUME DYNAMICS AND FLUCTUATIONS

3.1. Variation of Temperature with Downstream Position

It is often the case in the early stages of a plume's evolution that the initial momentum is more important than the acceleration due to buoyancy. In this case, in which momentum dominates the plume dynamics, one speaks of a *momentum plume*.³ The horizontally-directed Waxahachie plume certainly qualifies as a momentum plume.

The structure of a stack plume can be described in terms of three flow regions. Closest to the stack is the potential core or near field region, in which temperature and momentum are relatively flat functions of radial position and do not vary much with downstream position. Further out is the transition region, where the spread of the jet in the radial direction and the axial decay of the centerline properties are very rapid. Finally, there is the fully developed or far field region, where the radial spread is nearly linear with respect to axial distance, and the Gaussian plume model is applicable.⁴

With the use of appropriate conservation laws, hydrodynamic similarity conditions, and dimensional requirements, one can predict the behavior of plumes in the fully developed region.^{5,6} The velocity of a momentum plume is predicted to decrease linearly with downstream distance x. Since a momentum plume decelerates as it moves downstream, the average separation between plume particles in the x-direction *decreases* as a function of x. Because of this deceleration, the concentration σ and temperature excess above ambient fall off as only the first power of x:

$$\sigma \propto x^{-1} \tag{1}$$

$$\left(T - T_o\right) \propto x^{-1} \tag{2}$$

where T and T_o are the plume and ambient temperatures, respectively.

In the potential core region close to the stack, the plume temperature will be relatively constant as a function of downstream distance. Further out, the temperature should decrease in accordance with Eq. (2).

Confirmation of this behavior is seen in the published experimental data of Prengle *et al.*⁷ These measurements were taken on a power plant plume with an initial temperature of about 500 K, a flow rate of about 9 m³/s, and an initial upward velocity of approximately 11 m/s. There was no appreciable crosswind. Figure 2, derived from these data, shows the normalized temperature above ambient $(T-T_o)/(T-T_m)$ (where T_m is the maximum plume temperature) plotted against the reciprocal of the downstream distance. The 1/x temperature variation of Eq. (2) is seen for 1/x values below about 0.2 (i.e., for x > 5 stack diameters). This model seems to hold even for 1/x = 0.07 (i.e., for x as great as 14 stack diameters). At this downstream distance, the velocity had fallen to about 0.4 of its initial value. Evidently, at this velocity the plume was still acting as a momentum jet.



Figure 2. Axial temperature variation for power plant plume (adapted from Prengle et al. (Ref. 7))

Similar behavior is seen in the Waxahachie data. Figure 3 shows the temperature excess (degrees K over ambient) vs. the reciprocal of the downstream position (ft⁻¹). The temperatures shown are the maximum temperatures recorded by the thermocouple array in the plume at the given downstream distance (i.e. maximum "hot spot" temperatures during the total time at which the plume was at the given position). The nominal stack temperature for these runs was 130°C, although the measured temperature near the stack was considerably higher. Injection of ethanol at 200 ml/min had no noticeable effect on the plume temperature. The expected linear relationship is observed in the fully developed Gaussian region (left portion of the plot). For points with (1/Position) = 0.17 ft⁻¹ or less, the correlation coefficient is 0.93. The knee in the curve occurs at approximately 0.2 ft⁻¹, corresponding to a distance of 5 feet or 3.75 stack diameters. * This is consistent with the above-cited results from Prengle *et al.*, which show the knee at about 4 stack diameters.



Figure 3. Downstream temperature variation (Waxahachie thermocouple data).

For the Waxahachie plume, temperature varies linearly with position, if measured more than 6 feet from the stack. The temperature midway between Plane 1 and Plane 2 of the thermocouple array (the spectrometer location) will then be the average of the Plane 1 and Plane 2 temperatures.^{**} Less than 4 feet from the stack, the temperature should be essentially constant. In the transition region (between 4 and 6 feet), the temperature variation is more difficult to characterize.

3.2. Fluctuation of Plume Temperature

A spreadsheet macro was used to find the maximum plume temperature ("hot spot") at each time at which thermocouple data were recorded. For a typical data collection interval, the maximum thermocouple reading is plotted as a function of time in Figure 4. The thermocouple array position is graphed on the right axis of the figure. The thermocouple data show considerable variation, especially when the array is relatively far from the stack.

^{*} Waxahachie data taken with a hand-held thermocouple show the plume temperature falling off at distances of only 2 to 3 feet from the stack exit, in contradiction to the array data shown in Figure 3. This discrepancy may be due to a failure to locate the hand-held probe in the hottest part of the plume, and is an indication that plume meandering is significant in these experiments.

^{**} The spectrometer was positioned at the midpoint of the array, but the center of the plume cannot always be assumed to be visible at this point. As will be discussed in Section 3.2, the plume's horizontal position has considerable variation at distances greater than a few feet from the stack.



Figure 4. Maximum thermocouple readings (Plane 1 of array).

An animated color-coded display, showing thermocouple readings at Planes 1 and 2 for a user-selected time interval, was used to determine the position and temperature of the plume at times of interest. This animated tool provided useful information for the spectroscopic studies to be discussed in the following sections.

3.3. Plume Spectroscopy

Figure 5 shows a typical plume spectrum from the Waxahachie collection. Evidently, the plume absorbs background emission from the sky in the region from 9 to 13 μ m; this result is not surprising, since strongly absorbing water bands are present in this spectral region. Strong CO₂ emission is seen around 4.26 μ m and 15 μ m (cf. Ref. 1).



Figure 5. Waxahachie plume spectrum. Plume temperature 130°C; 1.52 m from stack exit.

3.4. Optical Depth and Modeling of Spectral Profiles

As radiation travels through a gas, it is subject to absorption. This process is quantified through the absorption coefficient $\alpha(\lambda)$, defined as the fractional amount of absorption per unit path length. This absorption is partially offset by emission of radiation by the gas volume being traversed. Since the emissivity is equal to the absorption coefficient, the rate of generation of radiation can be expressed as $\alpha I_{bb}(\lambda)$, where $I_{bb}(\lambda)$ is the blackbody spectral radiant emittance $(W \cdot cm^{-2} \cdot \mu m^{-1})$,

$$I_{bb}(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{ch/\lambda kT} - 1}$$
(3)

The intensity of radiation $I(\lambda, x)$ therefore evolves according to the equation⁸

$$\frac{dI}{dx} = \alpha I_{bb} - \alpha I = \alpha (I_{bb} - I)$$
(4)

If the initial intensity at position x_1 is zero, the solution is

$$I(x_2) = I_{bb} \left[1 - \exp(-\tau_{21}) \right]$$
(5)

where $\tau_{21} \equiv \tau_2 - \tau_1$ is the *optical depth* or *optical thickness* of the gas between positions 1 and 2. If the absorption coefficient is constant, the optical depth of a slab of gas of thickness L is simply αL .

When the optical depth is small, the intensity increases linearly with the optical depth (or with concentration, since α is proportional to concentration). At optical depths near unity, absorption has become significant, and the intensity is about 63% of the blackbody value. At large optical depths, the intensity approaches the blackbody value. In this case, where $\tau_{21} >> 1$ and the gas radiates at the blackbody limit, the gas is said to be *optically thick*. Such a gas is opaque; a photon at an optically thick wavelength has little chance of traversing the gas without absorption. Plume parameters deduced from optically thick spectral regions may reflect conditions in the outer layers of the plume, since emission from the central region may not reach the spectrometer.

The absorption coefficient, and therefore the optical depth, is a function of wavelength. A gas sample that is optically thick in the vicinity of a molecular transition will generally be optically thin in other parts of the spectrum. A dense gas will radiate at the blackbody intensity (which is itself wavelength-dependent) in spectral regions adjacent to transitions, but emit much less energy at other wavelengths. Thus, for a dense gas, the spectrum near an emission line will not show the shape predicted by quantum mechanics for single molecules. Rather, because optically thick emissions saturate at the blackbody value, measured spectra are often flat-topped near transition lines. Further away from the line center, the emission will eventually become optically thin, and "wings" will be seen with intensity predicted by the equations of quantum mechanics.

Using these basic concepts of radiation transfer, a simple optical transfer model was developed and used to explain the observed spectral shapes. In the model, radiation is assumed to be propagating along the y-direction. If the intensity of radiation at point y is known to be I(y), the intensity at point $y + \Delta y$ is, from Eq. (4),

$$I(y + \Delta y) = I(y) + \alpha(y)[I_{bb}(y, T) - I(y)]\Delta y$$

(6)

where the temperature T is a known (or assumed) function of y. The code finds the absorbance $\alpha(y)$, as a function of wavelength, by extracting values from the PNNL *Infrared Spectral Library*⁹ and adjusting for the position-dependent density. The plume CO₂ density is assumed to be a Gaussian function¹⁰ of the position coordinates x and z (note, however, that a Gaussian model may not be strictly appropriate for some of the spectra that were taken close to the stack, since the density in the near field region is expected to be flat-topped). The plume temperature excess above ambient is assumed to have the same Gaussian variation with transverse position. Results are smoothed to simulate the limited spectral resolution of the D&P spectroradiometer. The input parameters are wavelength range, plume radius, maximum plume temperature (at the plume center), maximum plume density, and location of the spectra obtained during the Waxahachie collection; agreement was very good.

As already noted, molecular transition bands actually consist of a number of closely-spaced lines, corresponding to transitions between rotational levels with various J (angular momentum) values.^{*} These individual lines are too close together to resolve with the spectrometers used in the plume studies reported in this paper. Therefore, for low-density plumes, the spectrometer will average a spectral region containing both peaks and gaps between peaks, and the resulting signal will be less than the peak intensities. This effect is illustrated in the model results of Figure 6, which shows the intrinsic spectrum of CO₂ in the vicinity of 4.26 μ m, with a simulated spectrometer signal.



Figure 6. Details of $CO_2(001) \rightarrow (000)$ spectral region, with simulated (smoothed) signal from D&P spectrometer. Low-density case.

If the gas density is increased, eventually the peaks of the spectral lines will reach their maximum intensity—the blackbody limit. In the gaps between the spectral lines the emissivity is small but nonzero. Hence there will be some radiation produced in these spectral regions, but at moderate gas densities this radiation will be below the blackbody limit. If this is the case, the spectrometer signal will not become flat-topped, even though the spectral peaks have saturated. The spectrometer signal is less than the blackbody limit, although it is greater, relative to the peaks, than the signal seen in the low-density case. At still higher gas concentrations, even the gaps between the closely-spaced emission lines will saturate at the blackbody limit. When this situation prevails, the limited spectrometer resolution is no longer a factor, and the spectrometer signal will be at the blackbody limit (Figure 7). The calculation of this spectrum assumes a CO_2 concentration-pathlength of 10,000 ppm-m, a value probably achieved or exceeded in at least some experimental plume studies.

Roughly speaking, optically thick plume emissions are limited by the temperature at the location where the gas becomes opaque (for the wavelength in question). In a plume that is cooler in its outer layers, *reversal* or *inversion* of spectral lines can occur.¹¹ As Eq. (4) indicates, if the intensity at a given wavelength is higher than the blackbody emission corresponding to the local temperature, there will be net absorption of radiation. In a moderate-density plume with this temperature structure, the central part of the flat-topped emission from an optically thick core will be partially absorbed, and the resulting spectral shape will show dips at wavelengths corresponding to maxima of the *R* and *P* branches. Further out on the spectral wings, where the absorbance is less, there will be little absorption.

^{*} For discussion of the quantum mechanics of molecules, see Ref. 1 and the references therein.



Figure 7. Details of $CO_2(001) \rightarrow (000)$ spectral region, with simulated (smoothed) signal from D&P spectrometer. High-density case (10,000 ppm-m). Plume temperature = 286 K (uniform).

These effects can be seen in Figure 8, which shows simulated plume spectra near the $4.26 \,\mu m \, CO_2$ line. The optical transmission computer model was used to produce spectra corresponding to the low-density "intrinsic" spectrum (a), the flat-topped spectrum when optical thickness becomes important (b), and absorptive cases when the concentration is increased still further [(c) and (d)]. Experimental observations of these phenomena are discussed below.



Figure 8. Modeled spectra in vicinity of 4.26 μ m transition. Spectrometer line of sight is through plume center. Maximum plume temperature at plume center = 600 K in all cases. All plots are to same scale.

3.5. Measured Spectral Shapes

In the earlier (2002) Waxahachie measurements reported in Ref. 2, the spectrometer was located a considerable distance from the vertically-directed plume, and the signal received at the spectrometer consisted of the high-*J* lines that are not affected much by absorption from the relatively cool CO_2 in the air. The optically thick part of the transition was removed by atmospheric absorption, and was therefore not observed. On the other hand, in the horizontally-directed stack measurements reported in this paper, the spectrometer was close to the plume. Hence, absorption by atmospheric CO_2 should be relatively unimportant in these recent Waxahachie data, and any absorption features probably originate from absorption by the dense CO_2 in the plume itself. The 2003 data should demonstrate optically thick conditions, unmasked by atmospheric effects. A close look at these data near the CO_2 transitions is therefore of interest.

Figure 9 shows a measured spectrum in the vicinity of 15 μ m (with the sky spectrum subtracted out). Apart from the effect of limited spectrometer resolution, the measured plume spectrum closely resembles the PNNL library spectrum shown in the figure, demonstrating that little absorption is occurring in this wavelength region (except possibly at the wavelength corresponding to the central Q branch spike). In the R and P branches, this transition is optically thin.



Figure 9. Measured CO_2 spectrum near 15 microns, with PNNL library spectrum. Plume temperature 130°C; 1.52 m from stack exit.

Near 4.26 μ m, on the other hand, absorption strongly affects the shape of the observed plume spectrum. In Ref. 2, CO₂ spectra in this wavelength region from the 2002 data collection were discussed. The spectra showed a strong absorption "hole" resulting from atmospheric absorption in the central part of the band. Beyond this absorption hole were the so-called red and blue spikes from high-*J* transitions that are relatively immune to absorption. The absorption removed the most intense part of the emission, where the conditions for optical thickness are satisfied. It was concluded that the high-*J* part of the spectrum that reached the spectrometer was optically thin, since the intensity of this signal was far below the blackbody limit. The shape of the observed spectrum did not depend significantly on plume temperature. These vertically-directed stack results exemplify the case where spectral absorption is primarily due to atmospheric CO₂.

Figure 10 shows horizontally-directed stack spectra (2003 data) for plumes of three different temperatures. The curves are normalized to comparable magnitudes. The spectra differ from the intrinsic spectrum of the CO_2 molecule (Figure 6) in shape, as well as in the wavelengths at which the spectral peaks occur. A central peak and an absorption dip around the central wavelength of the transition are present. These observations are qualitatively similar to the 2002 Waxahachie results. However, a significant difference seen in the recent data is that absorption is much more significant for data associated with a higher plume temperature. For cooler plumes, the spectral shape is relatively flat-topped.



Figure 10. Measured CO_2 spectra near 4.26 microns, at three plume temperatures.

Simulated spectra from the optical transmission model described in Section 3.4 can explain these results. Figure 11 shows simulated spectra for plumes with various central temperatures. These results show the same trend as the measured spectra of Figure 10: prominent red and blue spikes for high-temperature plumes, and a more flat-topped spectrum for low-temperature plumes. Hence, variations in central plume temperature can explain the experimental results. Physically, the hot central core of the plume, being optically thick, emits a flat-topped spectrum; absorption occurs as the emitted photons pass through the cooler outer region of the plume. This absorption is strongest at wavelengths corresponding to the maxima of the P and R branches. Cooler plumes show less of this absorption effect because they have less thermal contrast between the core and outer regions.



Figure 11. Effect of maximum plume temperature on modeled spectra. Spectrometer line of sight passes through plume center. Curves are normalized to comparable magnitudes. Maximum concentration (at plume center) = 5000 ppm.

However, a slightly more involved explanation is needed to explain the current data. Meandering of the plume can cause a low temperature along the spectrometer line of sight, even if the plume's central temperature at the downstream location of the spectrometer is high. Examination of the thermocouple animation sequences indicates that the "cool plume" data often correspond to runs in which plume meandering occurred. This scenario can also cause changes in spectral shape. Since the plume temperature increases as one moves inward toward the plume center, one can roughly regard the plume thermal structure as consisting of a hot core surrounded by an envelope of cooler gas. If the spectrometer line of sight passes through the plume center, it samples the full range of gas temperatures. Infrared photons from the emissive core must pass through the outer region of cooler gas, where they are subject to heavy absorption, to reach the spectrometer. High-temperature Waxahachie data generally correspond to this scenario. "Lowtemperature" data, on the other hand, often correspond to scans during which a hot plume veered to the side, and the spectrometer (and the corresponding elements of the thermocouple array) recorded conditions at the plume edge. In this case, there is little temperature variation along the line of sight, and the spectrometer sees a flat-topped spectrum, as would be expected from an optically thick gas at a single temperature.

The absorption of energy by a cool outer region was computed by the optical transmission model; results for various spectrometer lines of sight are shown in Figure 12. Clearly, sampling the plume along an off-center line of sight can cause the spectrum to appear more flat-topped, mimicking the "cool plume" spectra seen in Figure 11. These results suggest that, in some situations, the shape of the spectral region around the 4.26 μ m transition can be used to infer whether the spectrometer is viewing the central part or the edge of a plume. (For example, this explanation is appropriate if one knows the core temperature to be high at the spectrometer's downstream location; hence, changes in spectral shape can be attributed unambiguously to the instrument line of sight.) Due to plume meandering, it is often difficult to know what part of a plume was being sampled in a given measurement; hence, such a capability may be useful in studies that involve a spectrometer in close proximity to a plume.



Figure 12. Modeled plume spectra at various lines of sight, measured from plume center. Curves are normalized to comparable magnitudes. Maximum temperature = 600 K. Maximum concentration (at plume center) = 5000 ppm.

4. CONCLUSIONS

Measurements on the horizontally-directed plume demonstrate that considerable side-to-side motion occurs, even a few feet from the stack. However, by recording the maximum temperature during a measurement period, the expected relationship between temperature excess and downstream position was observed. Although random fluctuations are important, their effect can be minimized by appropriate experimental or analytical procedures.

Because the spectroradiometer used in this work does not resolve fine spectral structure, the measured emission does not necessarily correspond to the values of the peaks corresponding to rotational transitions, even if these peaks have reached the blackbody limit. In an extremely dense plume, on the other hand, even the gaps in between the peaks reach saturation. In this case, the signal is at the blackbody limit, and can be used to deduce plume temperature by matching the intensity to a blackbody curve. Of course, such a temperature determination may not give the maximum plume temperature if optical thickness prevents observation of the plume core region.

A previous attempt to model the shape of the CO_2 spectrum in the vicinity of the strong transition at 4.26 μ m was unsuccessful.² This problem arose because the central plume region was assumed to emit radiation with the intrinsic spectral profile of the CO_2 transition. However, plumes from combustion processes typically have high CO_2 concentrations, causing the emission from the plume core to be optically thick, and hence flat-topped, in this region. The optical transfer model described in this paper reproduces such optical thickness effects, and successfully models the spectral profile around 4.26 μ m.

The expected plume thermal structure – a hot core surrounded by cooler gas – was confirmed by measuring the shape of the spectral region near the 4.26 μ m CO₂ transition. With the assumption of a Gaussian plume structure, the radiation transfer model successfully reproduced the observed spectral shapes under various experimental conditions. Observation of such self-absorbed spectral regions offers promise as a diagnostic of the contrast between central and peripheral plume regions. Such measurements can provide insight into whether the central plume region was within the spectrometer line of sight.

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